



Stochastic Modelling of Landfill Directive Targets

Prepared for Defra

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Contents

1.	Introduction	1
2.	Model Structure	2
2.1.	Calculation of Diversion Capacity in 2020	2
2.2.	Calculation of Demand for Diversion in 2020	5
2.3.	Correlation Between Inputs	6
2.4.	Calculating Surplus or Deficit Capacity	8
3.	Model Results	9
3.1.	Assumptions Used For Model Calibration	9
3.2.	Illustrative Simulation Results	10
3.3.	Sensitivities	12
	Appendix A. Capacity Modelling Flowchart	17
	Appendix B. Using @Risk	18
B.1.	Conventions Used	18
B.2.	Static and Random Values	18
B.3.	@Risk Distributions	18
B.4.	Running a Simulation	19
B.5.	Advanced Analysis	20
B.6.	Correlation Matrix	24
	Appendix C. SARIMA Forecasts	25
C.1.	Implementing the SARIMA Model	25
C.2.	Understanding Conditional Predictions	26

1. Introduction

Defra commissioned NERA Economic Consulting to review its waste capacity modelling. Defra uses this modelling to assess whether England is likely to achieve the necessary waste diversion as part of the UK targets from the 2020 Landfill Directive. The target requires a reduction in Biodegradable Municipal Waste (BMW) consigned to landfill to 65% of 1995 levels by 2020, or 10.16Mt.

In an earlier report, we recommended that Defra adopts stochastic modelling techniques to better understand the likelihood of meeting the target.¹ In this report, we describe our subsequent work to develop a “Monte Carlo” simulation model, calibrated with some initial inputs, that Defra can develop for the purpose of conducting future waste infrastructure modelling work.

The model takes a range of inputs on future waste arisings and the supply of diversion capacity, while the main output of the model is a statistical distribution around the surplus or deficit BMW diversion capacity present in 2020. The model works as follows:

- The model takes forecasts of the most likely value of certain key input variables in 2020. These include waste arisings, commissioned diversion capacity, and the recycling rate (amongst others);
- The model also uses assumptions on the range of uncertainty around these key input assumptions, defined by assumed probability distributions. The model also allows for correlation between certain variables, reflecting the possible relationships that exist between them;
- The Monte Carlo model, implemented using the “@Risk” add-in to Excel, then recalculates surplus or deficit diversion capacity several using several thousand different randomly generated values from the assumed statistical distributions of the key input assumptions; and
- By combining the results of this large number of runs, the model derives a probability distribution around the surplus or deficit diversion capacity.

This report describes the model structure and the initial assumptions with which the model is currently populated (Chapter 2), and presents some illustrative results (Chapter 3). As noted further in the remainder of this report, the assumptions with which the model is populated are somewhat subjective and preliminary. Their primary purpose is for model calibration and testing.

¹ NERA Economic Consulting, “Review of Methodology for Forecasting Waste Infrastructure Requirements – Draft Report”, 28th September 2012.

2. Model Structure

As noted above, the Monte Carlo simulation model is implemented in Excel using the “@Risk” add-in software. The starting point for developing this model was the file submitted to NERA by Defra entitled “WIDP MC Forecast.xlsx”, into which we added the Monte Carlo simulation functionality.

In the sections below we describe the structure of the model’s inputs and outputs, and provide a practical guide to editing them in the spreadsheet model we have prepared.

- Section 2.1 outlines how to revise data and assumptions related to infrastructure;
- Section 2.2 outlines how to revise forecast data on waste arisings and account for new outturn data as it becomes available; and
- Section 2.3 outlines how to use the stochastic features of the Monte Carlo model.

2.1. Calculation of Diversion Capacity in 2020

2.1.1. Calculating available diversion capacity

The “supply” side of the model is diversion capacity in 2020. This capacity is a random variable in the “WIDP MC Forecast” model. The key technical assumptions are defined in the worksheets labelled “Infrastructure”. The key distributional assumptions are defined in the worksheet labelled “Inputs”.

The “Projects” worksheet adds up the effective capacity to divert BMW from landfill of all projects that are either online already, or expected to come online before March 2020. Since the effective capacity is the result of several factors that cannot be forecast with certainty, we have calculated it using randomly distributed variables that take on likely values for each key parameter. Diversion capacity for an individual plant is therefore calculated as follows:

BMW Diversion Capacity =

$$\text{Capacity} * \text{Delivery Dummy} * \text{Utilisation Rate} * \text{Efficiency} * (1 - \text{Share to EfW}) \dots \\ \dots * \text{BMW Content} * \text{Outages}$$

Where:

- “Capacity” is the operational capacity of the project in kt per annum, which is a uniquely specified input assumption for each unit, defined in the “Projects” worksheet (see Section 2.1.2);
- “Delivery Dummy” is a random variable that takes the value 0 or 1, as outlined in Section 2.1.2, with a probability specified by the user in the “Delivery Adjustment” worksheet. In each run of the simulation, the plant’s effective capacity is commissioned with this probability;
- “Utilisation Rate” is a percentage figure, typically less than 100%, which adjusts for the fact that plants do not usually run at their maximum reported capacity. Utilisation rates are a random variable, and are assigned a triangular distribution defined by the user in the “Inputs” sheet (see Section 2.1.3);

- “Efficiency” represents the percentage of the BMW sent to each plant that is ultimately diverted from landfill. Diversion efficiency depends on technical features of each diversion project, like moisture loss, recycling, and refuse derived fuels, among others. Users of the model can edit the technical assumptions associated with each MBT plant type in the “Diversion Efficiency” worksheet. These values are then assigned a triangular distribution in the “Inputs” worksheet (see Section 2.1.4);
- “Share to EfW” of each plant is an input assumption that can be defined by the user. In the case of EfW plants, this is zero. Other plant types send a certain proportion of their waste to EfW, where this share is a random variable with a triangular distribution defined by the “Inputs” worksheet (see Section 2.1.5);
- “BMW Content” is the percentage of waste diverted by each plant, and is calculated by multiplying the utilised capacity, in terms of Municipal Solid Waste (MSW), by the BMW content (%) of MSW. “BMW Content” of waste is a uniformly distributed variable, with a maximum and minimum value defined by the user in the “Inputs” worksheet (see Section 2.1.6); and
- “Outages” is a percentage, typically less than 100%, which accounts for the fact that commissioned plants may face unforeseen periods offline. This is assigned a triangular distribution in the “Inputs” worksheet (see Section 2.1.7).

In the following sub-sections we provide further information on these key input assumptions. The majority of these input assumptions can only be defined with reference to industry knowledge and expertise, and require a degree of subjective judgment. Hence, we defined the assumptions with which the model is populated through a series of discussions with Defra’s technical experts. While we believe that the working assumptions contained in the model are reasonable, there may be scope for refinement following further analysis, discussion or investigation. For this reason, the model provides future users with the flexibility to change any of the input assumptions described below.

2.1.2. Randomisation of project delivery

The “Projects” worksheet contains Defra’s list of waste diversion projects. This list comprises those plants that are already operational, and those that are due to come online before June 2020. These plants have four key inputs associated with them, which can be edited by the user in the worksheet:

- Operational capacity (kt);
- A commercial classification: Public Private Partnership (PPP), Private Finance Initiative (PFI), or Merchant (M);
- The financial year in which the plant is due to come online; and
- The type of plant – Electricity from Waste (EfW), Mechanical Treatment (MT), or Mechanical Biological Treatment (MBT).

For each project, a “Project RAG” is defined, where “RAG” stands for “Red, Amber, Green”. Each project’s RAG reflects its stage of development. For example, projects without financing or planning permission in place are flagged as “red”, whereas projects that are under construction are flagged as “green”.

From the RAG and the source of finance of each project, the “Projects” worksheet derives a “delivery adjustment rate” (DAR). The DAR is used in the model to define the probability that each project will come online, and is a number between 0 and 1.² Hence, the model uses the DAR for each project to randomly select which projects will be delivered. It does this by defining a uniform random variable between 0 and 1 for each project. If the random variable exceeds the DAR, the project is not commissioned in that run of the Monte Carlo simulation. If it is less than the DAR, then the project is commissioned. Therefore, a project with a DAR of 0.90 will be commissioned in nine out of ten iterations of the Monte Carlo model.

2.1.3. Utilisation rates

The model currently assumes that MT and MBT plants have a utilisation rate that varies between 75-90% according to a triangular distribution, with a modal value of 80%. Utilisation rates for EfW plants are also assumed to follow a triangular distribution, ranging from 90% to 105% with a modal value (i.e. the value that is most likely) of 100%.

2.1.4. Diversion efficiency

The model assumes that BMBT type plants have a modal diversion efficiency of 85%, bounded by a range of 70-90%. LFMBT type plants have a modal diversion efficiency of 76.8%, bounded by a range of 70-90%. EfW plants are assumed to be 100% efficient, and MT plants 0% efficient.

2.1.5. Tonnes to EfW

We assume that BMBT plants send 50% of tonnage to EfW, LFMBT plants send 0% of tonnage to EfW, and MT plants send 85% of tonnage to EfW. The model does not simulate random variation in the quantity of waste that each plant type sends to EfW.

2.1.6. BMW content of waste

The BMW content of MSW is assumed to remain at its assumed value of 68%, with no range of uncertainty currently included. The MSW content of industrial waste is assumed to vary uniformly between 18-20%, and the MSW content of commercial waste is assumed to vary uniformly between 83-85%.

2.1.7. Outage rates

The model applies a 95% adjustment to commissioned plants to account for the possibility of unforeseen outages. This assumption currently has no random variation associated with it, on the basis that, although outages might be spread randomly within each year, across the year as a whole there is less uncertainty associated with the total outage rates for each plant.

² DARs are specified as input assumptions on the “Delivery Adjustment” worksheet.

2.2. Calculation of Demand for Diversion in 2020

The “demand” side of the model is the level of residual BMW in 2020 (i.e. BMW not recycled or diverted from landfill). Forecasts generated outside of this model, as well as available data on outturn are entered by the user in the “Arising” worksheets.

As for our forecast of diversion capacity set out above, some input assumptions can only be defined with reference to industry knowledge and expertise, so the values with which the model is currently populated reflect our discussions with Defra’s technical experts. While we believe these assumptions are reasonable, the model provides future users with the flexibility to change any of the input assumptions described below if more data or information becomes available.

2.2.1. Calculating residual BMW

The model calculates residual BMW as follows:

Residual BMW =

$$\begin{aligned}
 & \text{BMW Content} * \dots \\
 & \dots [\text{HH MSW} * (1 - \text{HH Recycling Rate}) + \dots \\
 & \dots \text{Industrial Waste} * \text{MSW \% of Industrial Waste} * (1 - \text{C\&I Recycling Rate}) + \dots \\
 & \dots \text{Commercial Waste} * \text{MSW \% of Commercial Waste} * (1 - \text{C\&I Recycling Rate})]
 \end{aligned}$$

Where:

- “Household MSW” is derived from a SARIMA modelling process described in our previous report, that is calibrated using historic quarterly Local Authority (LA) data in order to produce mean (i.e. average) projections out to 2020.³ The errors around this mean projection are used to define the standard deviation of a normal distribution that we use to simulate random variation in household MSW through the Monte Carlo modelling. The parameters of this normal distribution are entered by the user in the “Inputs” sheet. Currently, the SARIMA model produces a forecast of household MSW in 2020 of 21.27Mt, with a standard deviation of 0.95Mt;
- “HH Recycling Rate” is the assumed rate of recycling achieved by households in 2020. Historical data is entered in the “Outturn” worksheet. Since future values are unknown, the user can define the modal (i.e. most likely) value and assign it a triangular distribution in the “Inputs” worksheet. The model currently assumes that the household recycling rate can range between 48% and 52%, with a mode of 50%;
- “Industrial Waste” is the forecast of industrial waste arisings, which is forecast by indexing current industrial waste arisings to forecast growth in GVA. A projected “efficiency saving” in waste per unit of GVA is also projected, as described in our previous report. “MSW % of Industrial Waste” is a percentage conversion figure that transforms industrial waste into MSW (c. 19%). Since this is uncertain, it is defined as a

³ We explain how to extract these numbers from the statistical programme STATA at more length in Appendix C.

uniformly distributed random variable with a range between 18 and 20% defined by the user in the “Inputs” sheet. This gives industrial MSW arising of 4.65Mt in 2020.

- The “Commercial Waste” term is the forecast of commercial waste arisings derived using a similar method to “Industrial Waste”. “MSW % of Commercial Waste” is a percentage figure that transforms commercial waste into MSW. This figure is typically much higher than for industrial waste (c. 84%). Since this conversion rate is uncertain, it is defined as a uniformly distributed random variables with a range between 83% and 85% defined by the user in the “Inputs” sheet. This gives commercial MSW arising of 21.91Mt in 2020.
- In practice, we add together C&I waste arisings as we apply the same recycling rate to both. This gives C&I MSW arising of 26.56Mt in 2020, which is the modal value of a triangular distribution with a maximum of 30Mt and a minimum of 24Mt.
- “C&I Recycling Rate” is the assumed rate of recycling achieved in these sectors by 2020. The model currently assumes that the C&I recycling rate can range between 55% and 65%, with a mode of 60%.

2.2.2. Shocks to waste arisings

Historical evidence suggests that household waste arisings have departed from their trend growth rates on at least one occasion, as we describe in Section 3.2.1 of our previous report. This suggests that our SARIMA model may understate the true uncertainty around household waste arisings in 2020, as it assumes that no further “structural change” occurs in the data series.

To allow Defra to model the effect of possible future shocks to the residential waste arisings data series, we have developed a function that allows the user to specify the probability that a shock will occur by 2020, and specify the magnitude of the shock in percentage terms. A random draw is then used in each simulation to determine whether the shock occurs or not. This can be specified for both positive and negative shocks separately. We have also incorporated the possibility of modelling random shocks to commercial waste arisings.

2.2.3. Model risk

In the case of household and C&I waste arisings, as we describe in Sections 4.1 and 4.2 of our previous report, a number of alternative forecasts are available. To reflect the risk that one forecasting method may be superior to another, we have included a function in the model that allows future users to (1) define a new forecast of C&I or household waste arisings, and (2) simulate randomly which of the new and existing forecast is selected for calculating the surplus/deficit of diversion capacity. Specifically, the user selects the percentage weight the model places on the existing (“central”) forecast, so that it is used x% of the time, and the new alternative forecast is used (100-x)% of the time.

2.3. Correlation Between Inputs

2.3.1. Possible justifications for correlation between random variables

An important part of assigning random distributions to different inputs in the model is to understand how the key sources of uncertainty might be correlated with each other. For example, there is a strong argument to suggest that waste growth in households and

businesses are positively correlated, as both may depend on macroeconomic cycles. The model we have developed allows the user to define correlation between these arisings forecasts by entering correlation coefficients between -1 and 1 in the “Inputs” sheet. For example, if the correlation coefficient between LA and C&I arisings is set at greater than 0, then @Risk will perform simulations in which both move together.⁴ This increases the range of possible outcomes.

Another example is possible correlation between waste arisings and rates of project delivery, on the basis that one underlying economic driver of diversion infrastructure development might be the total volume of waste arisings. If waste arisings in a certain area grow strongly, then infrastructure owners are more likely to receive Local Authority waste contracts to meet this growing demand for diversion. If the user defines the correlation between project delivery and waste arisings to be greater than 0, then @Risk will perform simulations in which greater demand for diversion is met with greater supply of diversion capacity, narrowing the range of possible outcomes.⁵

2.3.2. Assumed correlation coefficients

Since there is not enough historical data to estimate correlations, we have used subjective judgment to select plausible correlation coefficients in order to illustrate the effects of correlation between key variables as illustrated in Table 2.1.

**Table 2.1
Correlations Defined by NERA**

Input Variables	Coefficient
LA Arisings and C&I Arisings	0.25
LA Recycling and C&I Recycling	0.25
LA Arising and LA Recycling	-0.25
C&I Arising and C&I Recycling	-0.25
Arisings and Project Delivery	0.20
Projects and Projects	0.10

In the case of LA and C&I arisings, we think it is highly likely that these two sources of waste growth move together, since both are driven by similar underlying factors (i.e.

⁴ A correlation coefficient below zero would imply that an increase in one random variable is associated with a reduction in the other.

⁵ Care must be exercised in interpreting the coefficients relating to project delivery. Each project’s chance of delivery is governed by a separate uniform random variable, which is continuous, but the outcome is “discrete”. “Discrete” variables can only take a finite number of values (in this case, 0 or 1). Some levels of correlation can therefore not be established. For example, if Project A and Project B are negatively correlated, and Project B and Project C are negatively correlated, then Project A and Project C must move together as they move in the opposite direction to Project B. Project A and Project C cannot therefore be negatively correlated with each other, and the model does not allow the user to specify a negative correlation coefficient between project deliveries for this reason.

Therefore, the final two coefficients in the “Inputs” sheet are ordinal: larger values will increase the correlation of the random variables, but the exact number is not an accurate reflection of the simulated correlation. These coefficients actually proxy the underlying correlation that exists between the latent variables for each project which affect the observable binary outcome of project delivery. These include demand for diversion, location, and other factors affecting profitability. A formal treatment of this is not possible given the limited data available.

consumption, employment, etc.). We have therefore assumed a weak positive correlation between them. We also believe that the two recycling rates are likely to be driven by similar factors (e.g. environmental policy, public sentiment), and hence have also assumed a weak correlation between the two.

It is also plausible that, as waste arisings grow, the percentage of waste that is recycled will fall. This may occur, for example, because of exogenous limits on the quantity of recycling that can be undertaken. We have therefore assumed a negative correlation between the two recycling rates and their associated stream of waste arisings.

Finally, there is an argument to suggest that project delivery will be correlated with waste arisings, as described above, so we have assumed positive correlation between these two random variables.⁶ We have also assumed positive correlation between the chances of individual projects being delivered. The basis for this is an assumption that the underlying economic drivers that incentivise development of diversion capacity, which we have not attempted to simulate in this model, will to some extent affect all projects similarly. However, there is also an argument to suggest that if one project in a certain area is delivered then, other things being equal, competing projects in that area will not be commissioned. For that reason, we have restricted the correlation coefficient to be weakly positive in our initial assumptions, which can be revised by Defra to reflect alternative views or assumptions.

2.4. Calculating Surplus or Deficit Capacity

The “Output” worksheet summarises the analysis performed by the model. This is the only worksheet the user needs to access to run a simulation and access the model’s outputs. This worksheet calculates surplus/deficit diversion capacity using two methods.

- **Method 1:** Forecast infrastructure capacity in 2020 is subtracted from forecast residual BMW to give a BMW to landfill figure.
- **Method 2:** Baseline diversion is calculated in the financial year end entered by the user. This draws data from the “Outturn” sheet. To this, forecast incremental capacity additions in subsequent years out to 2020 is added. This gives the infrastructure capacity in 2020, which is subtracted from residual BMW as before.

These methods will typically return different distributions for surplus/deficit capacity in 2020, because they are based on different assumptions of the amount and performance of currently installed diversion capacity.⁷

⁶ The correlation between individual projects and waste arisings will not be equal to the correlation between total diversion capacity and waste arisings, which is an output of the model. This is reported in the “Empirical Correlations” worksheet.

⁷ See the appendix to our previous report for a discussion of these two alternative methods.

3. Model Results

As described above in Chapter 2, to calibrate and test the “WIDP MC Forecast” model, we have populated the workbook with a range of data provided by Defra and from our own analysis, which we detail in Section 3.1. We then present the results for modelled surplus/deficit capacity in Section 3.2. Finally, we demonstrate some sensitivity analysis to show the impact certain key assumptions have on the model’s performance in Section 3.3.

3.1. Assumptions Used For Model Calibration

Table 3.1 lists the distributional assumptions we have used on all the model’s key inputs. The basis used for these assumptions is described in more detail in the Chapter 2. These are initial suggestions, and we expect them to be revised in line with Defra’s views in each case. The model allows the flexibility to revise all of them.

**Table 3.1
Distribution of Random Variables**

	Distribution	Static Value	Parameter 1	Parameter 2
Waste Arisings				
LA Arising (Mt)	Normal	Mean 21.27 S. Dev. 0.95		
C&I Arising (Mt)	Triangular	Mode 26.56	Min. 24.00	Max. 30.00
Recycling Rates				
LA Recycling Rate	Triangular	Mode 50.0%	Min. 48.0%	Max. 52.0%
C&I Recycling Rate	Triangular	Mode 60.0%	Min. 55.0%	Max. 65.0%
Waste Composition				
BMW Content of MSW	Uniform	Static 68.0%	Min. 68.0%	Max. 68.0%
MSW % of Industrial	Uniform	Static 19.1%	Min. 18.0%	Max. 20.0%
MSW % of Commercial	Uniform	Static 84.3%	Min. 83.0%	Max. 85.0%
Utilisation Rates				
BMBT	Triangular	Mode 80.0%	Min. 75.0%	Max. 90.0%
EfW	Triangular	Mode 100.0%	Min. 90.0%	Max. 105.0%
LFMBT	Triangular	Mode 80.0%	Min. 75.0%	Max. 90.0%
MT	Triangular	Mode 80.0%	Min. 75.0%	Max. 90.0%
Diversion Efficiency				
BMBT	Triangular	Mode 85.0%	Min. 70.0%	Max. 90.0%
EfW	Triangular	Mode 100.0%	Min. 100.0%	Max. 100.0%
LFMBT	Triangular	Mode 76.8%	Min. 70.0%	Max. 90.0%
MT	Triangular	Mode 0.0%	Min. 0.0%	Max. 0.0%
Tonnes to EfW				
BMBT	Triangular	Mode 50.0%	Min. 50.0%	Max. 50.0%
EfW	Triangular	Mode 100.0%	Min. 100.0%	Max. 100.0%
LFMBT	Triangular	Mode 0.0%	Min. 0.0%	Max. 0.0%
MT	Triangular	Mode 85.0%	Min. 85.0%	Max. 85.0%
Adjustments				
Outages	Triangular	Mode 95.0%	Min. 95.0%	Max. 95.0%

Note: For variables such as “BMW Content of MSW”, all the parameters are set equal which indicates no random variation was assumed.

3.2. Illustrative Simulation Results

Using the initial assumptions presented above, we ran the model with 5,000 iterations to derive distributions around the surplus diversion capacity under both “Methods 1” and “2”, as the figures below illustrate:

- Figure 3.1 shows that using “Method 1”, the current Landfill Directive target of 10.2Mt is exceeded with a 98.7% confidence;
- In Figure 3.2 we present the results for “Method 2”, using outturn landfill return data from FY 2011 (4.32Mt) to establish the baseline diversion capacity. Using this method, the target is not missed in any of the 5,000 iterations;
- In Figure 3.3 we show the results using “Method 2” using outturn landfill return data from FY 2010 (2.53Mt) to establish the baseline diversion capacity. Using this approach, the model suggests there is a 97.9% chance of hitting the target.

These results show that under either approach there is a high probability of meeting the target, although the results from “Method 2” appear sensitive to the base year selected for the analysis

Figure 3.1
Method 1 - Target Met With 98.7% Confidence

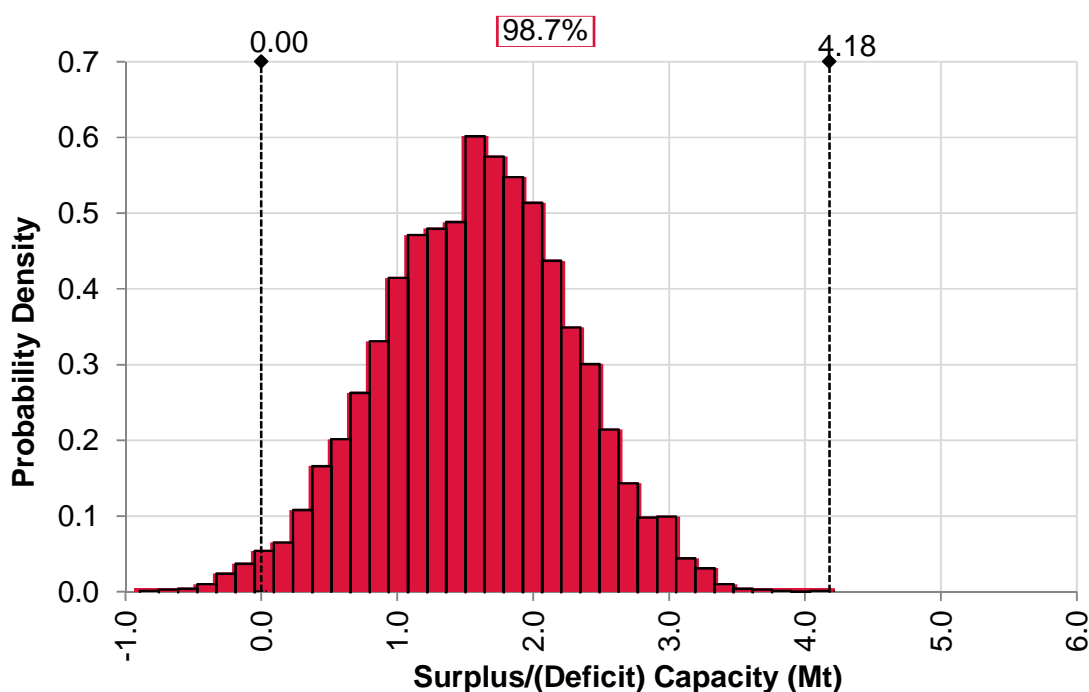


Figure 3.2
Method 2 (Baseline 2011) – Target Met With 100% Confidence

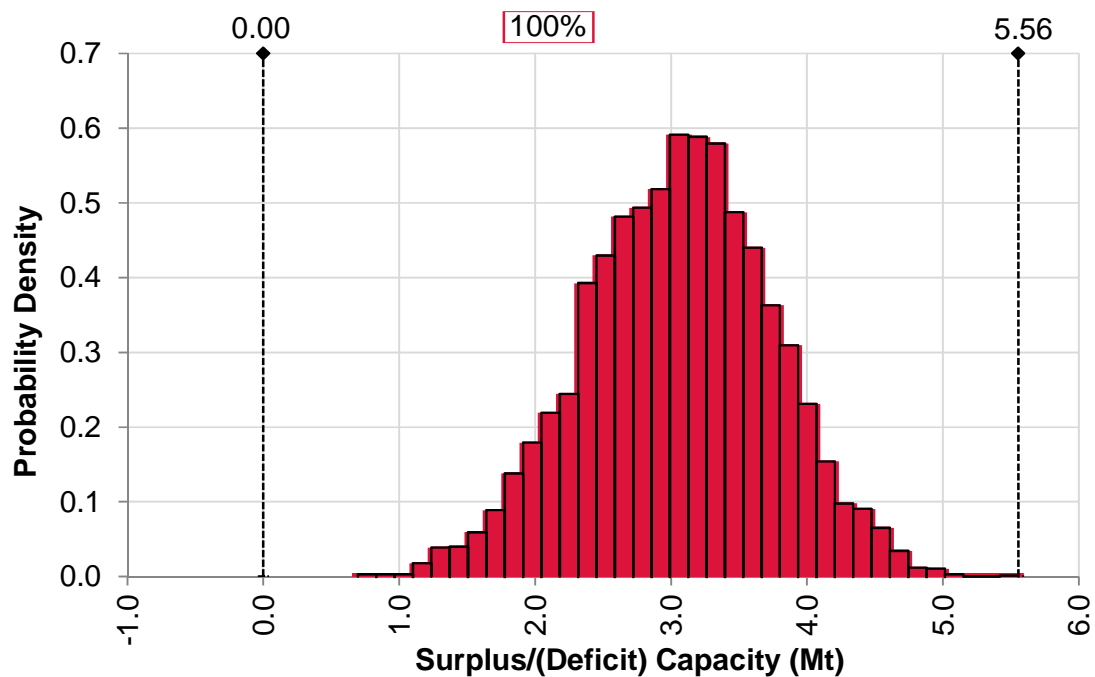
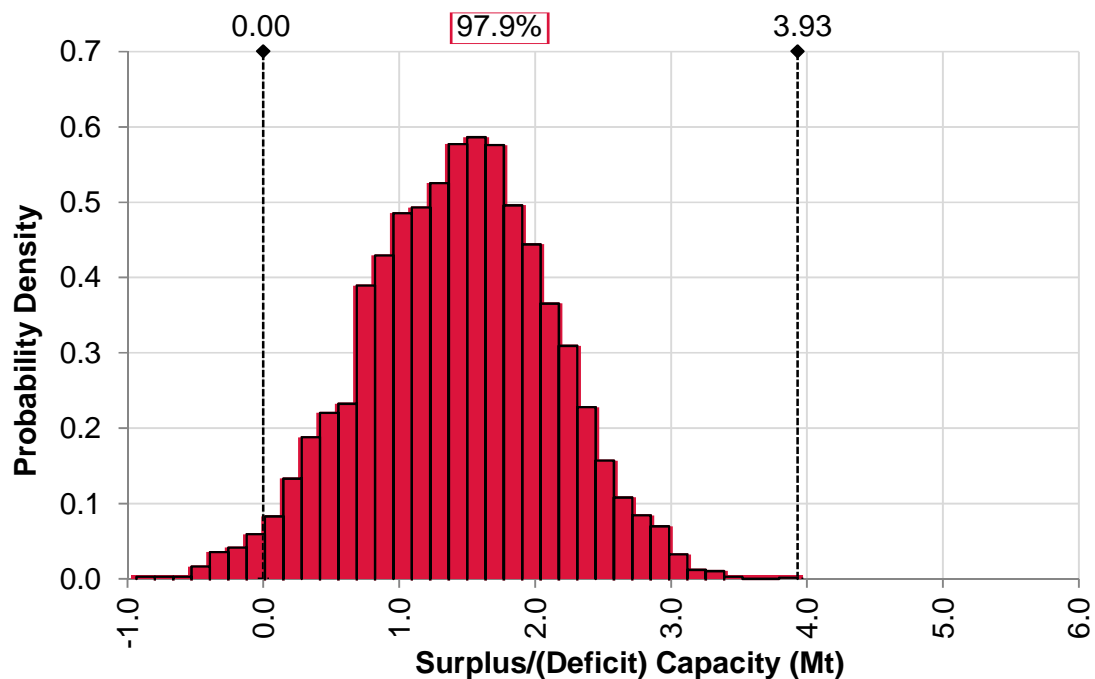


Figure 3.3
Method 2 (Baseline 2010) - Target Met With 97.9% Confidence



3.3. Sensitivities

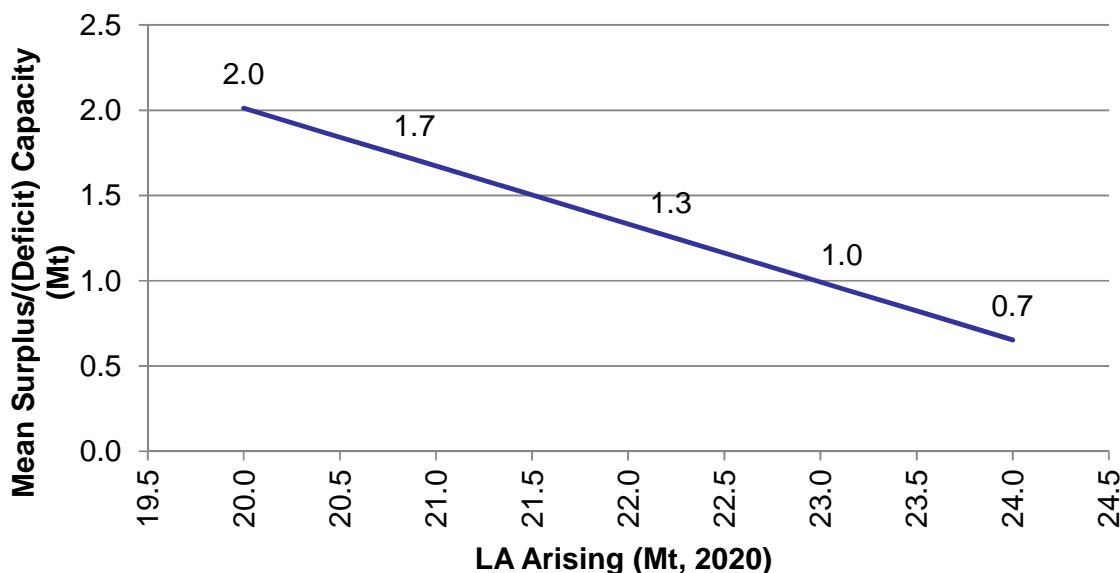
We have performed some sensitivity tests to examine how individual variables contribute to variation in the mean value of surplus/deficit diversion capacity. We have done so using @Risk’s “Advanced Sensitivity Analysis” feature, which we explain in Appendix B.5. Throughout this section we use “Method 1”, although similar results could easily be generated using “Method 2”.

3.3.1. Household waste arisings

To examine the sensitivity of the modelled output to a specific variable, we used multiple simulations. In the case of household waste arisings, we examined the impact of constraining this variable to take a specific value in each simulation. In the first simulation, LA Arisings was constrained to 20Mt in 2020, in the second 20.4Mt, increasing the value in steps of 0.4Mt each time.

Repeating this process, and recording the mean output each time, allows us to graphically examine the sensitivity of surplus/deficit capacity to changes in household waste arisings, as shown in Figure 3.4. The x-axis indicates the level of household waste arisings in 2020, whilst the y-axis represents the average value of surplus/deficit diversion capacity in 2020. As can be seen, a 2020 value of household waste arisings of 24Mt reduces surplus capacity to 0.7Mt on average.

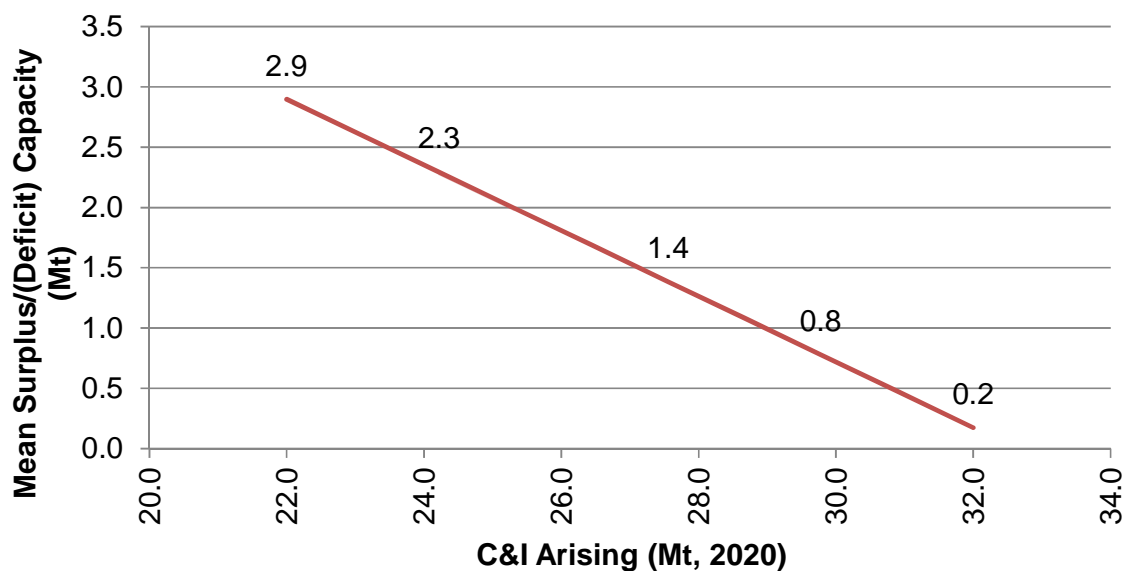
Figure 3.4
Sensitivity of Surplus Capacity to HH Waste Arisings



3.3.2. C&I waste arisings

We conducted a similar sensitivity analysis of C&I waste arisings, using the range of 22Mt to 32Mt. Figure 3.5 indicates that a 2020 value of C&I waste arisings of 32Mt distribution reduces the average surplus capacity to 0.2Mt.

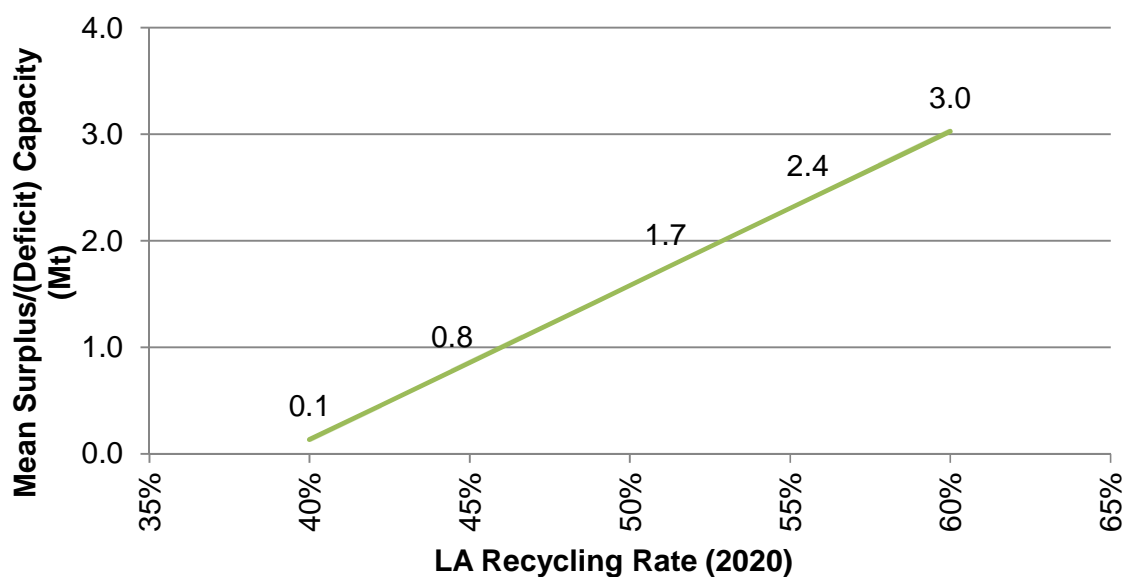
Figure 3.5
Sensitivity of Surplus Capacity to C&I Waste Arisings



3.3.3. Household recycling rate

We used a range of household recycling rates of 40%-60% to perform this sensitivity. If household recycling rates are as low as 40% in 2020 then surplus capacity falls to 0.1Mt on average in 2020.

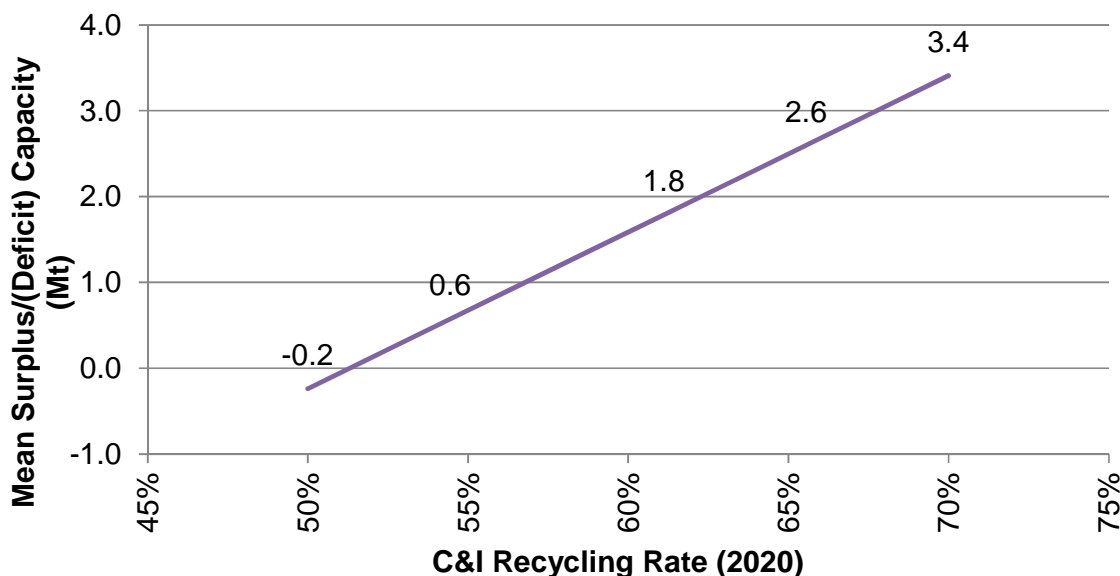
Figure 3.6
Sensitivity of Surplus Capacity to HH Recycling Rates



3.3.4. C&I recycling rate

The C&I recycling rate also has a large effect on outcomes, and constraining it to 50% in 2020 reduces surplus capacity to -0.2Mt..

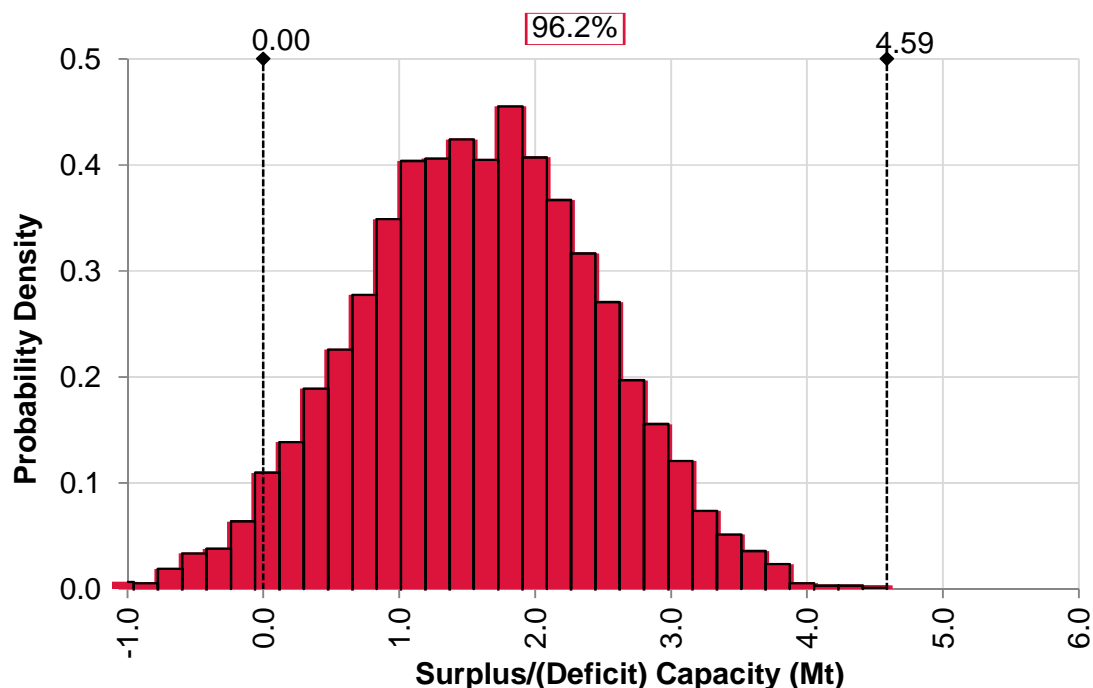
Figure 3.7
Sensitivity of Surplus Capacity to C&I Recycling Rates



3.3.5. Correlation between project delivery and arisings

We have also examined the effect of the correlations we assume in Table 2.1 by setting the correlation between waste arisings and project delivery to zero. Figure 3.8 shows that if we assume no correlation between waste arisings growth and project delivery, the probability of missing the target increases to 4%.

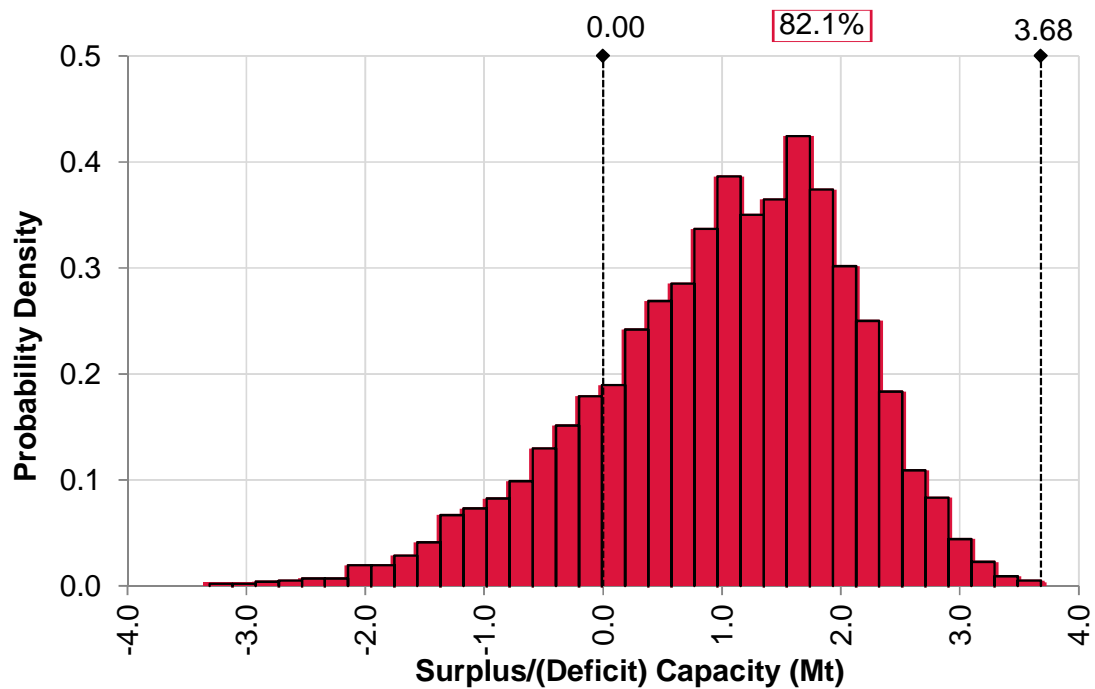
Figure 3.8
Zero Correlation Between Arisings And Delivery Reduces Confidence to 96.2%



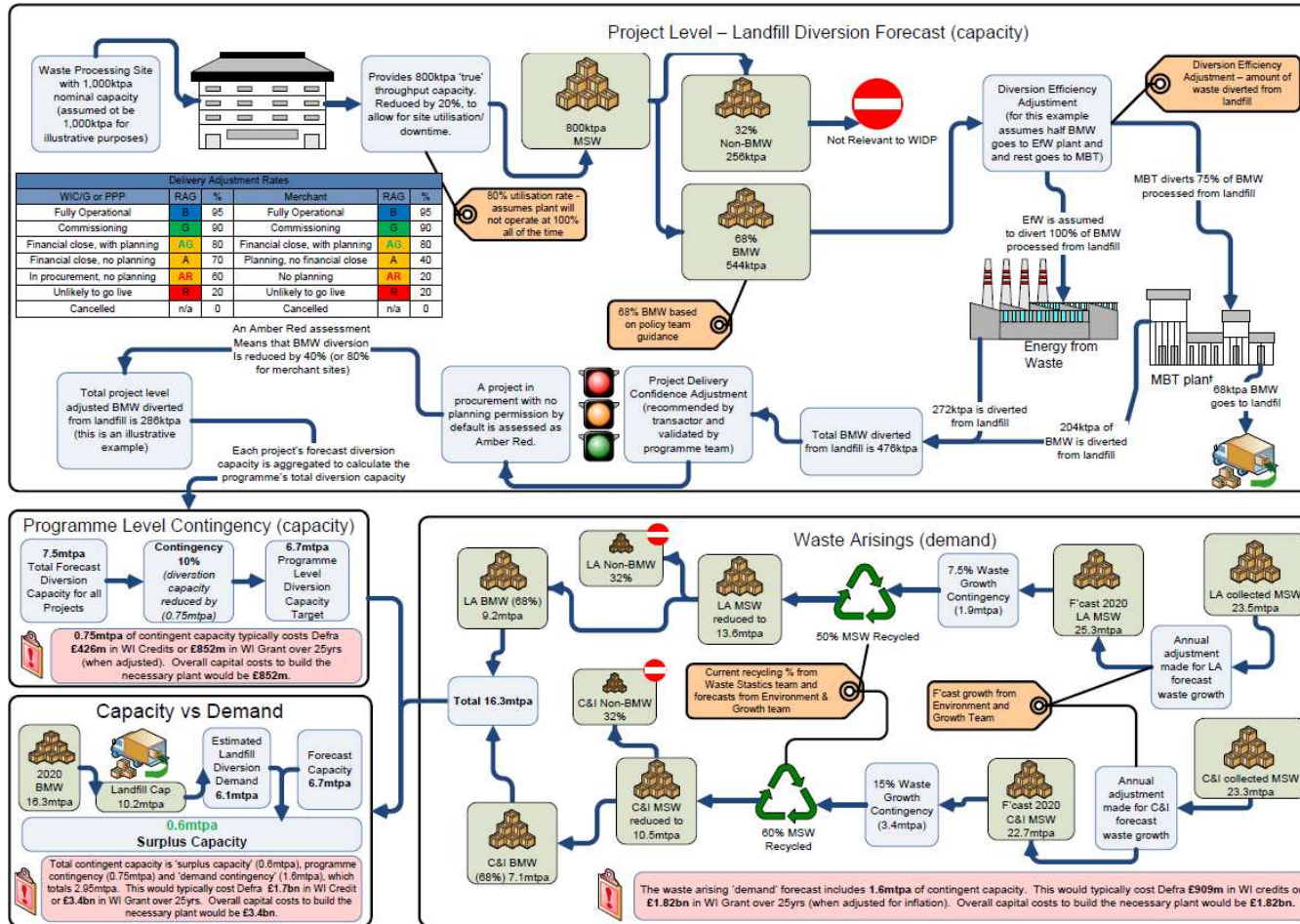
3.3.6. Shocks to household waste arisings

We have also tested the model’s sensitivity to allowing shocks to the forecast predictions of household waste arisings, as discussed in Section 2.2.2. For this purpose, we introduce two possible shocks, one to household waste and one to commercial waste, of +20%. We calibrate the model so that these two events each have an independent probability of 20% of occurring. As can be seen in Figure 3.9, this reduces the confidence with which the target is met to 82.1%.

Figure 3.9
Shocks to Household Waste Arisings Reduce Confidence Level to 82.1%



Appendix A. Capacity Modelling Flowchart



Appendix B. Using @Risk

B.1. Conventions Used

Throughout the model we have developed, a consistent convention is used to indicate the content of each cell. Observing this convention is useful to preserve the model's functionality.

Values in yellow are user inputs.

These define the distribution of the random variables and allow entry of historical data.

Values in blue are calculations.

Please do not edit these, as they include @Risk formulas.

Values in pale green are links to other cells or sheets.

Please do not edit these to avoid breaking links.

B.2. Static and Random Values

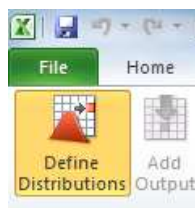
When the dice icon in the @Risk ribbon is toggled on (shown below, highlighted in orange) random variables in the model will take a random value. This means that, for values that have a defined probability distribution, or are affected by one, the actual number shown in Excel will only be one possible realization of many.



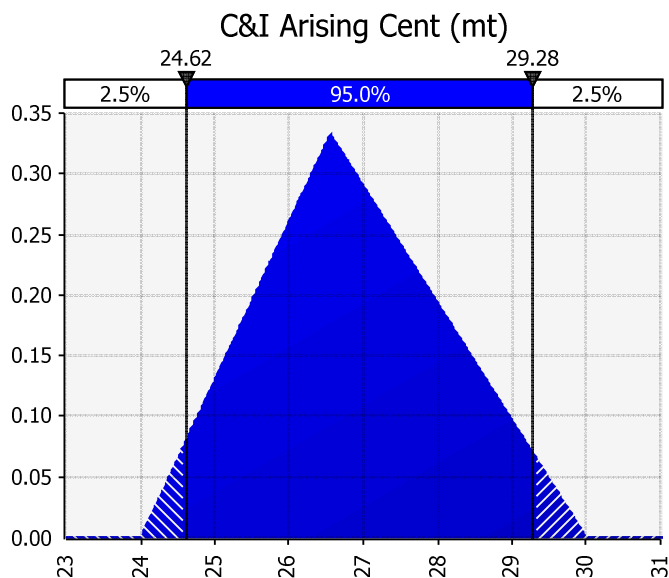
Toggling this feature off will cause random variables to take their “static” value, which in most cases is the average (mean or modal) value.

B.3. @Risk Distributions

To examine or edit the distribution of a model input, select a cell and click “Define Distributions” from the @Risk ribbon.



The distributions in the model are all already defined. Highlighting a cell and clicking “Define Distributions” will therefore give a graphical indication of the variable’s properties. Below, we show the triangular distribution for C&I waste arisings. This has a modal value of 26.56Mt defined by the forecasts, and a range of 24-30Mt defined by the user.

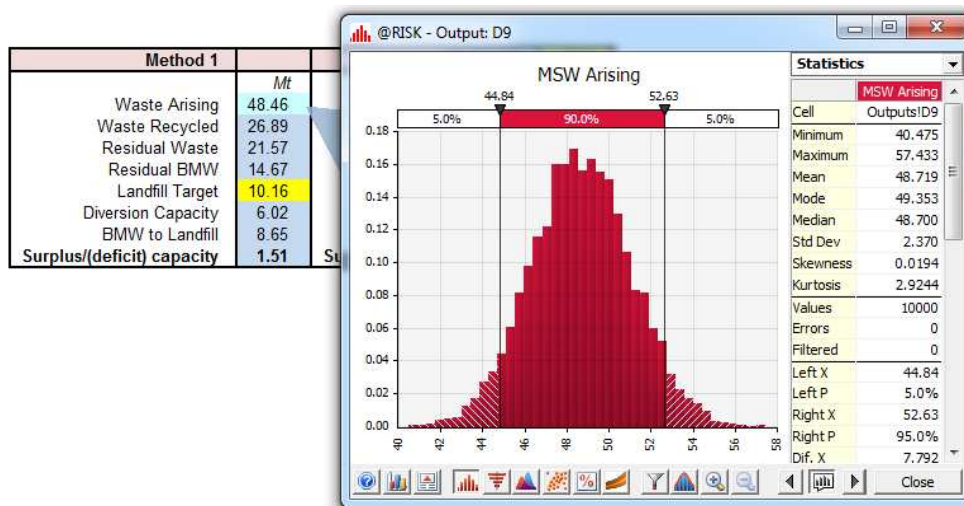


B.4. Running a Simulation

To run a simulation, first select the number of iterations from the @Risk ribbon. 1,000 iterations are sufficient for most purposes. Then click the “Start Simulation” button, shown below.



@Risk will now conduct the simulation, drawing random values for each input variable as defined by the user. Once this is done, it will present the results. Select a cell to see @Risk’s simulated distribution for that cell. In this example, we have selected “MSW Arising”.



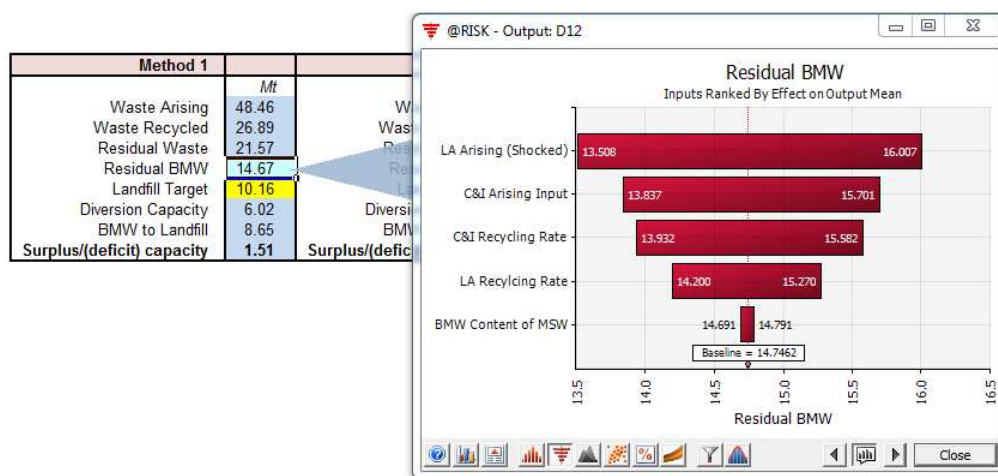
After a simulation has been run, this output can be viewed for any cell containing the formula “=RiskOutput()” by selecting the cell and clicking “Browse Results”. An overview of all the results can be viewed by clicking the “Summary” button, as shown above.

B.5. Advanced Analysis

@Risk has a very wide range of functions that allow the user to better understand the simulation results it produces. Here we give the example of two: “Tornado Graphs” and “Sensitivity Analysis”.

B.5.1. Tornado Graphs

After a simulation has been run, select a cell and click “Browse Results”. In this example we examine “Residual BMW”. Click the “Tornado Graph” button at the bottom of the displayed box.



This feature gives the user a quick insight into which of the model’s inputs most affect the output mean. In this case, random variation in the quantity of household waste arising, due to the distribution specified by the user, is shown to have a large effect on the random outcomes.

B.5.2. Sensitivity Analysis

Sensitivity analysis in @Risk takes the form of examining the consequence of one or more input variables taking values that are only assigned a very low probability. These are inputs from the 1st, 5th, 10th percentile of the distribution and so on. @Risk performs these sensitivities by running multiple simulations, each time constraining an input to take on only values drawn from a very low or high percentile.

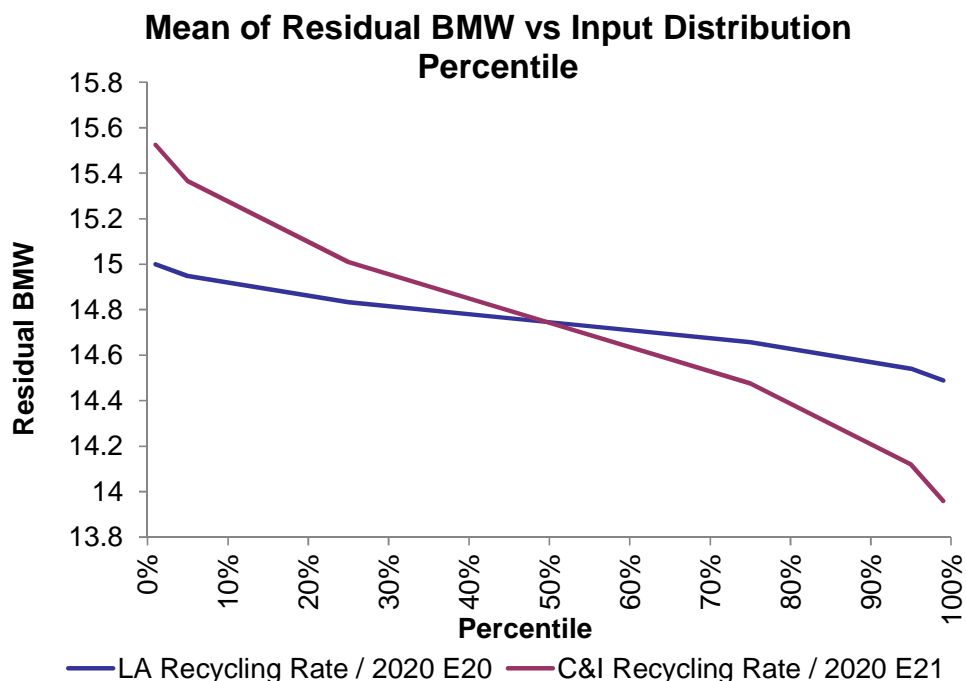
In this example, we examine the sensitivity of “Residual BMW” to LA and C&I recycling rates. First, select the number of iterations as before. Then select the “Advanced Analysis” button, choosing “Advanced Sensitivity Analysis” from the drop down menu.



The user will be prompted to enter a “Cell to Monitor” – this can be any cell with a “=RiskOutput()” formula. They are then prompted to “Add inputs” – these can be any number of cells that have an @Risk distribution. @Risk then performs several different simulations, and reports the results in a separate worksheet.

Here we present one of the outputs of the analysis, a graph that shows how the mean value of “Residual BMW Arising” varies with the percentile of the two random variables, “LA” and “C&I Recycling Rate”. This analysis shows that, given our assumed distributions, “Residual BMW” is more sensitive to changes in the “C&I Recycling Rate” in 2020.

Figure B.1
@Risk Sensitivity of Residual BMW to Recycling Rates

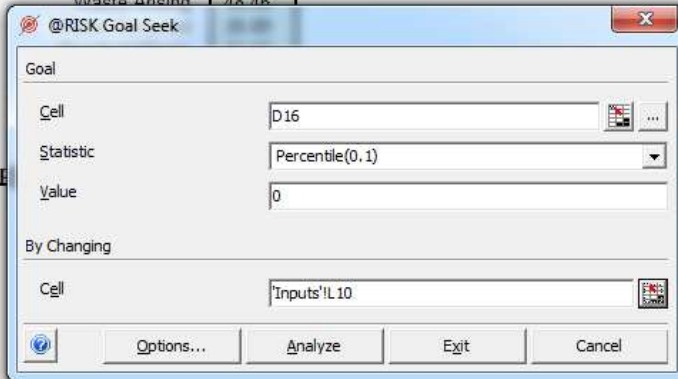


B.5.3. Goal Seek

@Risk allows the user to perform an analysis where a specific input is varied, in order for a specific output to achieve a pre-specified “goal”. In the context of the “WIDP MC Forecast” model, a useful feature is finding which level of inputs are necessary to achieve the 2020 target with a certain confidence level. @Risk runs multiple simulations, changing an input value each time to seek the goal the user has specified.

In this example, suppose the user wishes to see what maximum value of C&I waste arisings is consistent with meeting the 2020 target with 90% confidence. Select “Goal Seek” from the “Advanced Analysis” tab, and highlight the “Cell” to monitor. Here we have monitored surplus/deficit capacity. Select the “Statistic” for @Risk to achieve, here we have chosen the 10th percentile. Set the “Value” of the statistic, which in this case is 0.

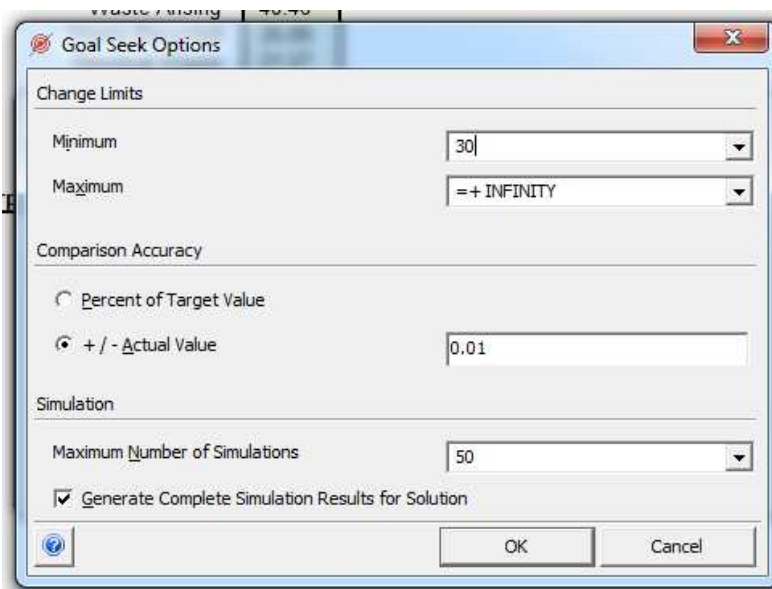
Method 1		Method 2	2011
	Mt		Mt
Waste Arising	48.46	Waste Arising	48.46
Waste Recycled	26.89		
Residual Waste	21.57		
Residual BMW	14.67		
Landfill Target	10.16		
Diversion Capacity	6.02		
BMW to Landfill	8.65		
Surplus/(deficit) capacity	1.51	Surplus/(deficit) capacity	1.51



The @RISK Goal Seek dialog box is shown with the following settings:

- Goal:** Cell: D16, Statistic: Percentile(0.1), Value: 0
- By Changing:** Cell: Inputs!L10
- Buttons: Options..., Analyze, Exit, Cancel

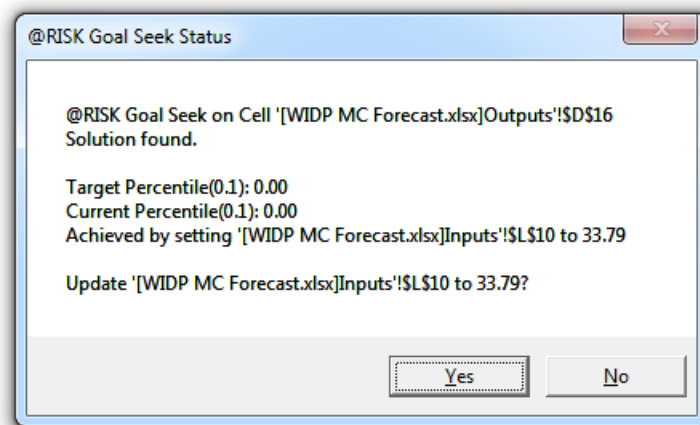
For “By Changing”, enter the cell which the user wishes to examine. This can be any cell that ultimately affects the output, and is not itself an @Risk output. This is true of the maximum value of C&I arisings. Select “Options” to call up some further options. In this example, we will only allow the value of C&I waste arisings to vary between 30Mt (its current value) and infinity. We will allow @Risk to stop the simulation when it achieves a value within 0.01 of the target. Finally, we will constrain the maximum number of simulations performed to 50.



The Goal Seek Options dialog box is shown with the following settings:

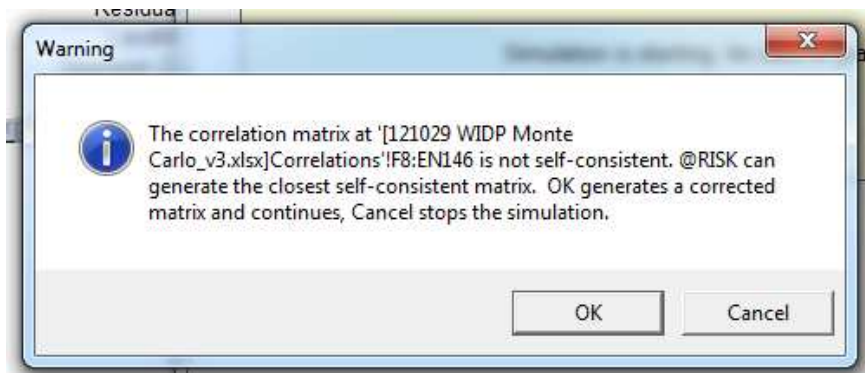
- Change Limits:** Minimum: 30, Maximum: =+ INFINITY
- Comparison Accuracy:** +/- Actual Value: 0.01
- Simulation:** Maximum Number of Simulations: 50, Generate Complete Simulation Results for Solution
- Buttons: OK, Cancel

After clicking “Analyze”, @Risk performs multiple simulations to seek this goal. It then presents the answer. In this case, if the maximum value of C&I arisings is raised to 33.79Mt then, using the model’s current assumptions, the target is met with 90% confidence. It asks the user whether to update the cell to this new value, which is usually not advisable.



B.6. Correlation Matrix

@Risk uses the matrix accessible in the “Correlations” worksheet to apply the correlations defined by the user. For large positive or negative values of correlation (close to +/- 1), the user may see the following message:



There are certain mathematical limitations placed on possible correlations. For example, if variable X and Y are correlated, and variable Y and Z are correlated, it is not possible for the correlation between X and Z to be zero.

@Risk will therefore adjust correlation coefficients entered by the user that are infeasible. We have developed the model so that these adjustments preserve as much of the original user entry for arisings and recycling data as possible, making changes to the correlation between project delivery. However, it is good practice to run the model again with adjusted coefficients.

Due to the complexity of the correlation matrix (139 rows x 139 columns), it is beyond the scope of this project to place analytical limits on the possible correlation coefficients that can be entered.

Appendix C. SARIMA Forecasts

In our earlier report, we provided results of a SARIMA model that forecasts household waste arisings using quarterly data from Local Authorities.⁸

C.1. Implementing the SARIMA Model

Alongside this report we have provided Defra with a file containing code for the statistical program called STATA. This file, “sarima_final.do”, will allow Defra to automatically update the forecasts we have generated when new information becomes available, by using the following procedure:⁹

- Save “sarima_final.do” in the same directory as an excel file containing the local authority data “LA_rawdata.xlsx”. We have provided an example of the formatting this file must contain;
- Open “sarima_final.do” in STATA and update the code to reflect the address where the rawdata is stored e.g. “C:\WIDP\Model\LA_rawdata.xlsx”;
- Edit the date at which out-of-sample predictions are made. For example, if Q2 2012 data is available, update all instances of “dynamic(tq(2012q1))” to contain “2012q2”.
- Select “File>Do” from the menu system in the STATA “.do file editor”. This will initialize the estimation, and save the data to an Excel file called “LA_final.xls” in the same directory; and
- Open “LA_Final.xls” and the data is presented as shown in Table C.1. Enter the prediction for 2020 and the standard deviation into the “WIDP MC Model”.

Table C.1
Output Contained in "LA_final.xls"

FY	SARIMA	Outturn	Std_Dev
2008	0.00	25.29	0.00
2009	24.72	24.33	0.00
2010	23.31	23.67	0.00
2011	23.17	23.45	0.00
2012	23.12	22.90	0.00
2013	22.85	0.00	0.72
2014	22.66	0.00	0.90
2015	22.40	0.00	0.94
2016	22.18	0.00	0.95
2017	21.95	0.00	0.95
2018	21.72	0.00	0.95
2019	21.50	0.00	0.95
2020	21.27	0.00	0.95

⁸ See Section 3.2.2. of our previous report.

⁹ To test that the model is still the best fit to the data, we have provided Defra with a file called “sarima_tests.do” which runs certain specification tests on the data.

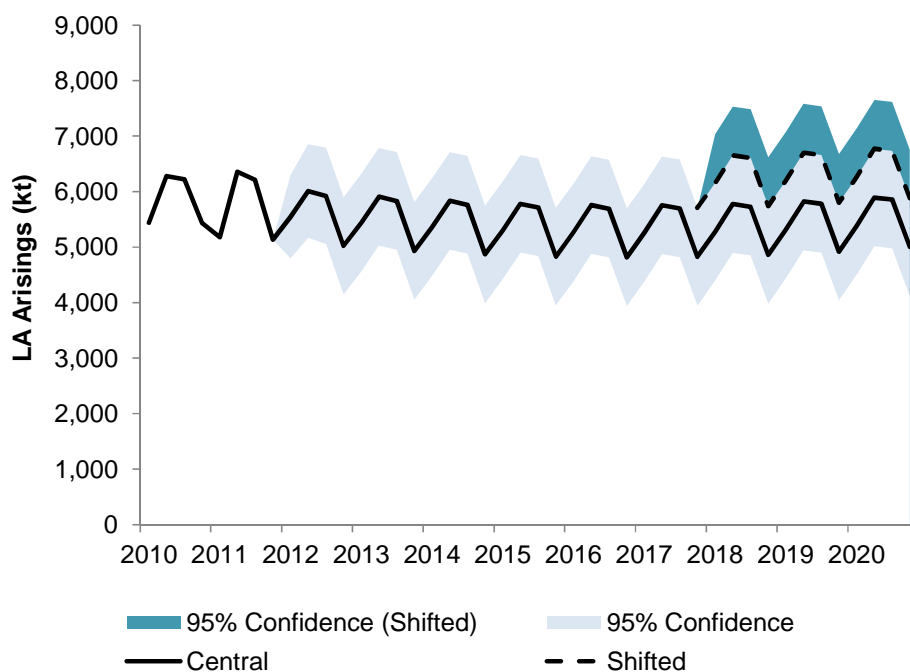
C.2. Understanding Conditional Predictions

No regression model making “out of sample” predictions can ever give an exhaustive picture of the likely outcomes in 2020. The standard deviation associated with each forecast period gives a level of uncertainty surrounding each prediction, but this standard deviation is calculated using data observed historically. Hence, while the forecasts emerging from the SARIMA model reflect the expected range of future waste arisings, this range may alter as more information becomes available over time.

This occurs because the SARIMA model’s projection of future waste arisings is a function of (1) past observations of waste arisings, and (2) random shocks. As future random shocks will occur between now and 2020, the expected range of 2020 waste arisings will also change. For instance, Table C.1 illustrates that an upward random shock to the waste arisings series occurring in Q1 2018 could materially affect the expected distribution of waste arisings for 2020.

Hence, the model should be updated regularly to both check its performance, and adjust the distributions forecast for 2020. The possibility of large one-off “shocks” can also be examined using the functionality described in Section 2.2.2 of this report.

Figure C.1
Forecasts Values Are Conditional On Currently Available Information



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